Grade 3 Mathematics Lesson Plan

June 24
G 3

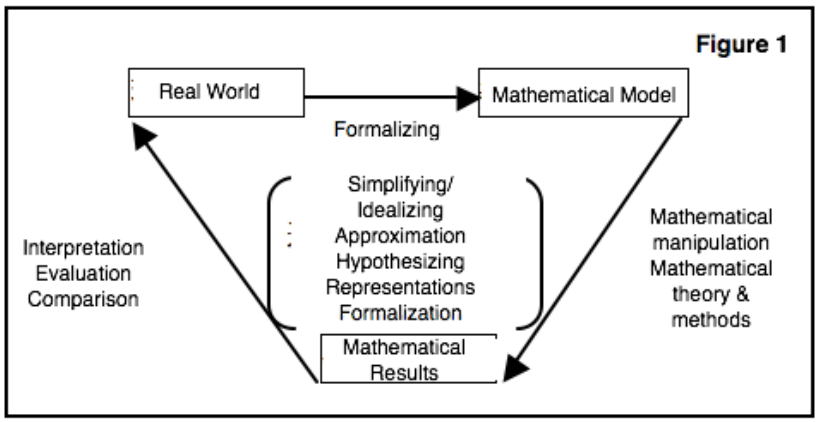
Date: June 24, 2017
Yamanashi University Attached Elementary School
Grade 3, Classroom 3 (35 students)
Teacher: NOMURA, Miyoko

Mathematics
Division with Remainders

1 About the Unit

"Understanding" and "doing correctly" are critical considerations in the elementary school mathematics teaching and learning. It is important that we teach mathematics in such a way that students can sense they come to undersatnd and become able to carry out procedures correctly. In addition, we we look ahead, another important idea is that students can "use" mathematics. What is needed in today's mathematics education is to nurture the disposition, "I can use mathematics. I am going to use mathematics," in our students.

When we think about the question of how we can teach mathematics so that students will feel that they can "use" mathematics, we realize that one critical idea is for students to be able to understand what they learned in mathematics classrooms mean in the real world. In other words, it is critical that students can interpret mathematical results in the real world. About this process of interpreting mathematical results in the real world situations, Miwa (1983) labeled Mathematical Modeling Process and represented in the diagram shown in Figure 1. The step of interpreting a mathematical result in the real world in this Mathematical Modeling Process is evident in the topic of today's lesson, interpreting the quotient and the remainder and reach the answer that is one more than the quotient. In this lesson, students will be asked the question, "How many boats are needed if 23 students are to ride 4-person boats?" Students will calculate "23 ÷ 4" and obtain the answer of "5 remainder 3." Then, they must interpret the result of the calculation and come up with the answer of "6." This is the situation in which students understand that the results obtained from calculations must be interpreted in the problem contexts and come up with the answer that is different from the calculation results. By tackling this type of situations that are challenging to students and helping them understand it, we want students to feel that they can "use" mathematics.



It is anticipated that the topic in this unit, “Division with Remainders,” may be difficult for some students as they get confused which of the numbers obtained in calculation should be considered as the answer. Moreover, a

challenge in teaching this topic is how students should interpret the meaning of remainders. In addition, in the 2016 National Assessment at Grade 9, the success rate for the problem, "When a number is divided by 3, the quotient is *a* and the remainder is 2. Express the number in terms of *a*," was only 33.6 %. This results shows that many students have difficulty in grasping the relationships among numbers and quantities and expressing them using mathematical equations. These results show that even though students could express the relationship of numbers that is shown as division with a remainder using the inverse operation of multiplication in elementary schools, their success rate declines significantly when variables are used to represent the same relationship. Thus, it is important that, in elementary schools, students develop a deep understanding of the relationship among the dividend, the divisor, the quotient and the remainder.

Students learned about division in Grade 3 for the first time. Up to this point, students' learning of division has been based on their everyday experience of "sharing," and they formalized it as a mathematical idea and learned to represent those situations using calculation expressions. Moreover, they have also learned that there are two types of division situations, fair sharing division and measurement division, and also division might be considered as the inverse operation of multiplication. In teaching division, we have made sure that it went much more than just studying computational methods. Instead, we have intentionally incorporated activities with concrete materials, drawings and diagrams to represent own reasoning processes so that students can deepen the concept of "sharing" mathematically.

The goal of this unit is for students to be able to apply division with remainders in problem situations just as much as division without remainder. The division problems discussed in this unit are those that can be solved by using the basic single-digit multiplication in reverse. Later in Grade 3, students will learn about division with larger dividends only in those cases without remainders. These experiences will be necessary for their study of division using the algorithm in Grade 4, and therefore, it is essential that students muster the necessary skills as well. Therefore, we aim to deepen students' understanding of the meaning of the division operation as well as realizing the meaning of remainders in division situations. We will emphasize the meaning of the dividend, the divisor, the quotient and the remainder in a division equation as numbers in a mathematical expression always have meanings. Moreover, we also want students to think about the results of calculations in the context of the original problems. Our aim is for students to be able to interpret the relationship between the quotient and the remainder obtained from calculation and the problem context correctly.

2 Goals of the Unit

- ◆ Students will deepen their understanding of the meaning of division by understanding division with remainders, and they can apply their learning in other situations.
  - ○ Students try to make sense of the meaning of and method of calculating division with remainders by relating it to division without remainder, multiplication and manipulation of concrete materials. (Interest, Eagerness, and Attitude)
  - ○ Students develop a unified understanding of division whether or not there is any remainder, and they can represent its meaning and method of calculation using concrete materials, diagrams, and mathematical expressions. (Mathematical Way of Thinking)
  - ○ Students can carry out the calculation of division with remainders and determine the quotients and the remainders correctly. (Mathematical Skills)
  - ○ Students understand the meaning of division such as the meaning of remainders and the relationship between the remainder and the divisor. (Knowledge and Understanding)

3 Relationship between the Unit and the Research Theme

(1) About the dispositions/abilities we want to nurture in this unit

In the mathematics group, in order to realize lessons in which students create their own mathematics, we utilize lessons that focus on problem solving (*mondai kaiketsu gakushu*). In the learning processes in problem solving lessons, the four types of dispositions/abilities and "questions" are closely related.

In the stage of grasping the learning task, students will have the question, "What have I learned so far?" and put the problem situations from their daily lives onto the mathematical playing field as the first step of problem solving. In the independent problem solving stage of the lesson, students will ask themselves, "Which of what I have learned may be useful in this problem?" and tackle the problem by comparing it to previously solved problems. However, it is not always possible for students to have their own ideas. Thus, their peers will become an important component of their learning. During the comparison and critical reflection stage, students of compare and contrast their own ideas with those of their peers to generate better solutions, approaching the goals of the lesson. In this stage, students will ask about the rationale, commonality, differences, and generalizability. During the reflection stage, students will ask about the merits and extendability of ideas so that they can use what they learned in other situations. In mathematics, we believe the engine for learning is "question." In the process of learning, when one problem is solved, a new "question" arises. "Questions" are continuously generated. What support this learning process is students disposition to tackle problem solving autonomously.

In this unit, the main question is to think about the meaning of division with remainders. Specific questions such as the following are anticipated: “Even when we cannot divide evenly, can we represent the problem situation using division equation as we learned previously?” and “What does the remainder represent in the problem situation?” Through examining such questions, we want to help students deepen their understanding of the meaning of division. We want to nurture the four types of dispositions/abilities in our students while pulling out questions such as “Why is it?” “How can we solve it?” and “Can we use the method of calculating division we learned previously?” as they encounter situations involving division with remainders.

(2) About strategies for “lessons in which students feel the values of learning”

In the mathematics group, we consider “values of learning” is realized in experiencing the merits of mathematics. The strategies necessary for “lessons in which students feel the values of learning” are as follow:

① Strategies to devise lessons in which students have questions and the sense of expectations
- Set up the learning tasks that generates “questions.”

② Strategies to devise lessons in which students are engaged independently and with each other
- Investigate “questions” that must be asked.
- Anticipate how a series of “questions” may be generated.

③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction
- Record “questions” on blackboard and reflect on the learning.
- Secure time for students to write reflective journal entries.

In this unit, we focus on helping students grasp their everyday experiences of “sharing” as a mathematical idea of “division.” Then, by putting division back in everyday situations,

we want students to be able to apply it in their daily life. In order to do so, we need to set up the learning task that is easy for students to relate to as well as interesting enough to hook them to the task. Problem situations that are easy for students to relate to and imagine are easier for them to represent their reasoning using diagrams and words. We want to conclude the task by developing a shared understanding by representing their diagrams and words as mathematical equations.

Moreover, we will pay close attention to the nature of questions so that we will have a series of questions. Throughout the unit, from the initial questions such as “Even when we cannot divide evenly, can we think of it as division?” and “Can we use the method of calculating division we learned previously?” we want to set up tasks so that students may ask questions such as “Can we still use multiplication to check the calculation of division with remainder?” and “How should we deal with the remainder as we answer the problem?” In addition, we want to make sure to record those questions on the blackboard, and we will encourage students to organize their notebooks so that we can look back on the flow of their learning in a lesson as well as the unit as a whole.

As students write reflective journal entries, we will encourage them to write specific ideas. In each lesson, we want students to develop the habit of recording what questions they had, how they resolve the question using their prior learning. By having a record of accumulated changes, we want students to have the sense of achievement. Furthermore, we want students to use their own reflective journal entries from previous lessons as motivation for new “questions” in the following lessons.

(3) About methods for assessing the quality of individual student’s learning

As a strategy to assess the quality of individual student's learning, we will make use of their notebooks. We have been encouraging students to make their notebooks align with the process of problem solving lessons, " grasping the learning task → independent problem solving → comparison and critical reflection → reflection." Thus, in the independent problem solving stage, students will write their own ideas. In the comparison and critical reflection stage, they try to record their peers' ideas. When they do so, instead of simply writing down the answers, we have encouraged them to include the steps and process of getting the answers, using words, pictures, diagrams and mathematical expressions. By writing their learning journal entries, students can organize their ideas, reflect on them deeply and make use of the ideas in new problem situations. This way, students can reflect on the development of their ideas. By checking students' entries in independent problem solving/comparison and critical reflection/learning journal against each other, we will know what ideas students initially had, what challenges they faced, and how their ideas evolved. In this way, we want to assess our efforts to increase the quality of individual student's learning.

In this unit, we design lessons so that students will develop the ability to express their ideas related to the meaning of equations and expressions in their notebooks. In addition, we want to assess students’ ability to connect problem situations with the meaning of division as well as their ability to effectively use their prior learning. Moreover, we plan to provide in-class support so that students might express changes in their own ideas and also similarities and differences with their friends’ ideas. We plan to provide necessary support so that students will understand useful ways of observing problem situations in order to find solutions.

4 Unit Plan and Assessment (Total of 10 lessons)

#	Goals	Learning Activity	Assessment Standards
① Division with remainders (6 lessons)			
1	Students will understand ways to calculate division with remainders in the case of 1-digit divisors and 1-digit quotients.	○ Think about ways to find the answer for 14÷3. ○ Learn that 14÷3=4 rem. 2.	[Interest] Students are trying to figure out ways to calculate division with remainders based on their prior knowledge of division. [Thinking] Students can explain ways to calculate division with remainders based on their prior knowledge of division using concrete materials, diagrams and equations/ expressions.
2		○ Lear the meaning of remainders. ○ Learn that even when the dividend cannot be divided evenly, we can still use the ideas of division.	
3	Students understand the relationship between the divisor and the remainder.	○ Explore the relationship between the divisor and the remainder in the case of 13÷4.	[Knowledge] Students understand that the remainder must be less than the divisor.
4	Students understand that division with remainders can apply in the case of fair sharing division situations.	○ Set up the calculation expression, 16÷3, based on the understanding of the problem context and think about ways to find the answer.	[Thinking] Students can explain ways to calculate division with remainders in the fair sharing situations based on their prior understanding of fair sharing division using concrete materials, diagrams and equations/ expressions.
5 6	Students understand how to check their calculation of division with remainders.	○ Think about ways to check division calculations, including situations with remainders.	[Knowledge] Students understand ways to check division calculation even when there are remainders.

② Problems that require the meaning of remainders (2 lessons)			
7 Today's Lesson	Students will deepen their understanding of how to think about remainders.	○ Based on the understanding of the problem situation, think about the reason why the answer for the problem should be Quotient+1 when the calculation is 24÷3=5 rem.3. ○ Think about other dividends for which the answers are "6 boats" based on the idea of inverse operation.	[Knowledge] Students understand how to deal with remainders based on problem contexts.
8		○ Based on the understanding of the problem situation, think about whether the answer to the problem should be the quotient as it is or Quotient+1 when the calculation is 30÷4=7 rem. 2.	
③ Unit Summary (2 lessons)			
9	Students can create problems while thinking about the meaning of division as well as how remainders should be treated.	○ Create problems that reflect different meanings of division.	[Knowledge] Students can solve problems using what they learned in this unit.
10	Consolidate students' understanding by applying their learning in various situations.	○ Engage in application problems and deepen their understanding.	

5 Today's lesson

(1) Date: June 24, 2017 (9:00 - 9:45)
(2) Location: Yamanashi University Attached Elementary School

(3) Goal of the lesson

- Students will deepen their understanding of remainders, and they can obtain answers to problems by rounding up the remainder, if necessary.

(4) Reason for teaching this lesson

Through the previous lesson, students have been exploring ways to calculate division with remainders, how to calculate, and using the inverse operation to check the calculations. Students understand that in both fair sharing (partitive) and measurement (quotative) situations, we can use division to represent situations when we are making equal groups even if there are remainders. They know that even when there are remainders they can use multiplication to find the quotients and use the inverse operation (multiplication) to check the results of calculation. However, students can easily fall into believing they "understood" division with remainders when they master the calculations. Therefore, by engaging in this lesson, students will think about the meaning of division and what the quotients and remainders represent, which in turn will deepen their understanding of division with remainder.

Today's problem is set in the context of a school excursion. As we think about the boat ride at the amusement park, they will be asked the question, "All 23 of us will ride the boats. Each boat can carry 4 people. How many boats will we need?"

In one of the research lessons in 2015, we presented a research lesson titled, "Let's think about division: Situations to think about remainder (1)" (Ohma). In that lesson, just as in today's lesson, students tackled the problem involving 23 ÷ 4 = 5 rem. 3." They then have to think about the problem context and decide how to answer the original problem, rounding up the remainder if necessary. For the 2015 research lesson, we anticipated that students might have difficulty answering the problem with "6 boats" by rounding up the remainder based on the problem context. Therefore, we hypothesized that by examining the calculation process to check division, students will understand the reason for rounding up the remainder to answer the real world problem. However, in that lesson, it was observed that some students clung to the quotient of "5" they obtained through calculation and had difficulty answering the problem with "6 boats." Although the calculation, 4 × 5 + 3, is the calculation to check the answer, 5 remainder 3, but it was also noticed that some students thought it was the calculation to check the answer to the problem, 6 boats (Seino, 2015).

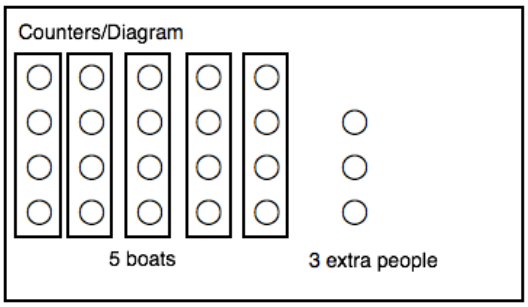
The goal of this lesson is to help students understand how to deal with quotients and remainders based on the problem context. It is anticipated that those students who use the calculation result as the answer to the problem will respond with "5 boats with the remainder of 3 students." In addition, since the question asks students "how many boats," some might ignore the remainder and simply answer, "5 boats." However, the problem statements says "all 23 students will ride the boats." Therefore, students will realize that they need a boat for the remainder of 3 students, thus affecting how they answer the problem. Through this experience, we hope that students can deal with remainders appropriately.

In today's lesson, students will be asked the question, "What are other possible number of students who can ride with 6 boats?" Based on the fact the situation, 23 ÷ 4 = 5 rem. 3, can be represented using the inverse operation, 4 × 5 + 3, students will think about what are other

possible number of students for whom we need 6 boats, and think about when we have to round up the remainder to answer the problem. In today's situation, there are 4 cases when we need 6 boats: ① 4 × 5 + 1 = 21, ② 4 × 5 + 2 = 22, ③ 4 × 5 + 3 = 23, and ④ 4 × 6 = 24. Thus, they notice that the cases when we need 6 boats are when the number of students are 21 through 24. In that proces, we will emphasize "4 × 5 = 20" means "4 students in each boat, and 5 boats will make the total of 20 students." Students will then notice that if we have 1, 2, or 3 more students than 20, we will need 6 boats. Thus, by going through the process of interpreting mathematical results in the context of the real world, we want to help students to overcome the challenge of coming up with the answers by rounding up the remainders. We also hope that students will deepen their understanding of division with remainders and consolidate their understanding of how to deal with remainders based on the problem contexts.

(5) Flow of the lesson

min.	Main learning activity/content Anticipated responses	• Points of consideration • Strategies for "lessons in which students feel the values of learning"
5 G R A S P	1. Grasp today's problem We are all going on the boat ride. Each boat can carry 4 people. How many boats do we need? • Grasp the problem situation. ○ It's division. • What if there is a family of 4? ○ They can ride the boat. ○ They need 1 boat. • What if there is a group of 8 people? ○ They can ride on the boat. ○ They can if they have another boat. ○ They can split into 2 groups of 4. So, they need 2 boats.	• Display a picture of 4-passenger boat so that students can grasp the problem context more easily. • By considering the situations with 4 and 8 people, we try to help students imagine the problem situation and develop a plan to tackle the problem. • By considering the situations with 4 and 8 people, make sure that students understand that there are more than 1 boat. **In the process of "lessons in which students feel the values of learning"** ⊙ Strategies • Motivate students to tackle the problem by helping them grasp the problem situation accurately. • Set up the learning tasks that generates "questions." • Write down students' "questions" on the board and help them understand a division equation can represent the problem situation.

10 T H I N K	2. Independent problem solving ○ Use their prior learning, students will find the answer. a) Use counters. b) Draw diagrams  c) Use multiplication $4 \times \square = 23$ Since 23 is not in the 4's table, we will have a remainder. $23 \div 4 = 5$ rem. 3 d) Use the quotient and the remainder as the answer to the problem, 5 boats with 3 people remainder. e) Because there is a remainder, not sure what the answer is. f) Use only the quotient of 5 and answer "5 boats." g) Realize that the 3 students left over must be on a boat, so the answer is "6 boats."	**In the process of "lessons in which students feel the values of learning"** ⊙ Dispositions/abilities we want to nurture in this unit • Think about whether or not we can use a division to represent the problem situation by clearly understanding the problem situation and thinking logically. • Observe students carefully so that the teacher has good sense of how students are using the strategies (a) ~ (c) and obtaining the answers (d) ~ (g).
25 D E E P E N	3. Compare and contrast solutions. • Share how many boats are needed e) 5 boats with 3 people remainder. f) I was able to do the calculation, but I'm not sure what the answer is. g) "5 boats." h) "6 boats."	• We will discuss the solutions in the order of (e), (f), (g) and (h). • Share all the ideas and think about what's different and why. • Discuss cases where the answer is incomplete, like (f), and clarify what it is that might be creating the challenge. • Ask students to raise their hands to show which one matches their answers.

	"Even though the calculation results are all the same, why do we have different answers?" • Comparing the differences in answers, and discuss how many boats are needed. ○ (d) and (g) are similar, but one includes the remainder of 3 and the other one does not. ○ In (h), the answer is "6," and that's different from the calculation result of "5 remainder 3."	**In the process of "lessons in which students feel the values of learning"** • In order to solve the problem, explore the task using mathematical thinking. ⊙ Students who feel the values of learning • Using the prior learning, students can represent the problem situation using diagram, words, and equations.
	"I wonder if we can explain using diagrams or counters?" (a) and (b) ○ It's the same with counters or with diagrams. ○ There are 4 people in each boat, and there are 5 boats.	• Share the idea of the students who used counters as a class.
	"Let's represent in an equation." • Think about how different ideas that have been shared can be represented by an equation. ○ 23 ÷ 4 = 5 rem. 3 ○ 4 × 5 + 3 = 23	**In the process of "lessons in which students feel the values of learning"** ⊙ Students who feel the values of learning • Students realize that they can explain their ideas clearly to others by using mathematical expressions.
	"Is '5 remainder 3" an answer?" ○ "5 remainder 3" is not an answer. ○ "remainder 3" does not match "how many boats do we need?"	
	"What should we do about the remainder of 3?" ○ We want the 3 remaining students to be on a boat, too. ○ Is it ok to use one boat even though there are only 3 people? ○ If all of us are going to enjoy the ride, we can't leave them out. ○ We need one more boat. ○ I think we need 6 boats.	• Make sure students understand where the 3 remaining students come from and the fact that they are not on a boat. • Because students' ideas might have changed from the beginning of the discussion time, ask students what their answers are and record the numbers.

	• Listen to the explanation of students who are not convinced "6 boats" is the answer, and discuss. ○ The part inside the box (in the diagram) shows 5 boats. ○ I understand we need 5 boats.	• If any students is still clinging on the answer of "5 boats," or if anyone is not convinced "6 boats" is the answer, ask them to explain why they feel that way. Then, discuss how many people can ride on 5 boats.
	"If there are 5 boats, how many people can ride?	• By confirming to what "5 boats" corresponds, help students understand the quotient of 5 corresponds to "5 boats" and how 5 boats are represented in the diagrams or counters. • Help students recognize "4 × 5" in the diagrams or counters. • Make sure students understand that "4×5=20" means that 20 people can ride the boats. Draw out the idea that 3 more students need to get on a boat.
	"How many people can ride on 6 boats?" • Think about the necessity of having 6 boats based on the inverse operation, i.e., multiplication. ○ Since there are 4 people on each boat and there are 6 boats, 4 × 6 = 24, or 24 people. ○ Since we can represent it as ☐ ÷ 4 = 6, ☐ must be 24. ○ 24 people can ride the boats if there are 6 boats.	• Ask students who think the answer is "6 boats" to explain the reason why they think so. Then, discuss the number of people that can ride 6 boats. **In the process of "lessons in which students feel the values of learning"** ⊙ Strategies • Draw out students' own "questions" from the task. "What should we do about the remainder of 3 people?" "Why do we need 6 boats?
	• Based on the fact that 20 people can ride on 5 boats and 24 people on 6 boats, represent the number of people between 20 and 24 using division or the inverse operation. Think about when we need 6 boats. ○ For 20 people, 20 ÷ 4 = 5 5 boats 4 × 5 = 20	• By thinking about other cases where we need 6 boats, verify that we get the answer, "We need 6 boats," from the calculation of 23 ÷ 4. • Sometimes in addition to the calculation results, we must pay

	○ For 21 people, 21 ÷ 4 = 5 rem. 1 6 boats 4 × 5 + 1 = 21 ○ For 22 people, 22 ÷ 4 = 5 rem. 2 6 boats 4 × 5 + 2 = 22 ○ For 23 people (today's problem) 23 ÷ 4 = 5 rem. 3 6 boats 4 × 5 + 3 = 23 ○ For 24 people, 24 ÷ 4 = 6 6 boats 4 × 5 = 24 • Think about how many boats are needed if there are 35 students (the number of students in our class). ○ 35 ÷ 4 = 8 rem. 3 Therefore, 9 boats. ○ 4 × 8 + 3 = 35 Therefore, 9 boats. ○ If there is any remainder, we need one more boat to put everyone on the boats. ○ Even if we have many more people, the remainder means one more boat.	attention to the remainder to answer the problem. • Reflecting on their prior learning of the relationship between the quotient and the remainder, from without remainder, that is "remainder 0," to "remainder 3" are the cases where we need 6 boats.
	5. Reflect on today's lesson ○ Sometimes we must consider the remainder to answer the problem. ○ We cannot ignore remainders. • Write reflective journal entries.	**In the process of "lessons in which students feel the values of learning"** ⊙ Methods for assessing the quality of individual student's learning • Through students' reflective journal entries, assess students' reasoning processes. • From what students wrote in their notebooks, assess how their thinking changed and whether or not they truly understood the reason why the answer is 6 boats.

(6) Points of observation

- Were the strategies to help students feel the values of learning?
  ① Strategies to devise lessons in which students have questions and the sense of expectations
  ② Strategies to devise lessons in which students are engaged independently and with each other
  ③ Strategies to devise lessons in which students can experience the sense of achievement and satisfaction bt reflecting on the lesson

(7) References

Omitted