June 25

Grade 9 Mathematics Lesson Plan

Date: Tuesday, June 25, 2019
Koganei Lower Secondary School
Attached to Tokyo Gakugei University
Grade 9 Classroom C (40 students)
Teacher: SHIBATA, Sho

1 Name of the unit: Variables and equations/expressions
[This unit includes the discussion on functions that are proportional to x^2, quadratic equations, and square roots]

2 Goals of the unit

- Students will understand the basic concepts and theorems/properties of square roots of numbers, polynomials, quadratic equations, functions proportional to x^2, etc. At the same time, they will master skills to mathematatze phenomena, interpreting things mathematically, processing and representing ideas mathematically, etc.
- While paying attention to the range of numbers, students will develop and nurture the capacity to examine the properties and calculations of numbers, to investigate relationships among numbers and quantities and their properties, to represent those relationships and properties, and to examine properties of functional relationships by relating tables, equations and graphs.
- Students will realize joy and merits of mathematical activities and develop the disposition to think persistently and make use of mathematics in everyday life, to reflect on their problem solving processes and try to evaluate and improve them, and to recognize diverse ideas and seek better ways to solve problems.

3 About the unit

(1) Flow of the unit

In elementary schools, students used the symbols such as □ and △ in equations such as 5 + □ = 8 and 3 x △ = 24, to grasp the relationship between addition and subtraction or multiplication and division. They also learned to express and interpret relationships among numbers and quantities using equations with words, such as (speed) x (time) = (distance). Furthermore, as the foundation for the study of algebraic expressions and equations in lower secondary school, they learned to use letters such as *a* and *x* in place of □ and △ in equations, and used equations to grasp and represent direct and inverse proportional relationships. They also used the idea of quasi-variables as they thought about ways to calculate division by fractions. The idea of quasi-variables is also utilized to represent properties and patterns among numbers. They have many experiences of representing a relationship between two quantities using a single number and identifying characteristics and patterns of two co-varying quantities.

Even in our daily life, there are many situations where we control (or make judgment) one of two quantities that are changing simultaneously (or they are predicted to be changing simultaneously). For examples, based on the cost of a train ticket, we might think “we’ve come a long way,” or we might hear a news story that “today’s youths have lower physical capacity” based on the data about students’ softball toss as an indicator for students’ “physical capacity,” which cannot be seen.

In Grade 7, based on students' learning in elementary school, students learned about the role of letters as variables, and explored the ways to manipulate and calculate with algebraic expressions and equations. In addition, instead of simply determining an unknown number, starting with the examination of quasi-variables (numbers that are treated like a variable), students learned to interpret the relationship that is represented by an equation or to represent and generalize the relationship using algebraic equations. Then, based on the examinations of everyday phenomena, students were introduced to the concept of functions, and re-examined direct and indirect proportional situations as examples of functions and investigated the relationships among equations, tables and graphs.

In Grade 8, students learned to use more letters in algebraic equations, which allowed them to examine a wider variety of phenomena. At the same time, they learned that they must think carefully about the relationship among quantities when 2 or more variables are being considered. For example, when trying to explain and justify the statement, "the sum of two consecutive even numbers is an even number," they must think about how to represent the two even numbers. They must decide whether to represent the two even numbers by using an integer n, as ($2n$, $2n + 2$), or using 2 integers n and m, ($2n$, $2m$). A model response would be ($2n$, $2n + 2$), but since the representation ($2m$, $2n$) involves less restrictions, justification with this representation does actually prove the given statement. However, the fact that the sum of any two consecutive even numbers is double of the odd number in between the two even numbers can only be see from the representation, ($2n$, $2n + 2$) – because the sum can be expressed as $2(n + 1)$. This is an example of how the advance in students' capacity to use algebraic expressions and equations leads to their capacity to reason at a higher level. It is also a good opportunity to illustrate how reasoning and representations are mutually beneficial. Students then examined everyday phenomena using the idea of linear functions.

Although the discussion above focused on topics in the domains of expressions/equations and functions, we try to pay attention to reasoning with functions even in the area of geometry. A typical example may be when students examine the quadrilaterals obtained by connecting the mid-points of the sides of any quadrilaterals. Determining what conditions relate to the special conclusion and how is indeed an application of reasoning with functions. In this way, we have been teaching students to reason with functions in many different units.

In Grade 9, this study continues with the topics such as factoring, polynomials, quadratic and cubic functions and equations. As we designed the unit which deals with these topics, factoring, polynomials, quadratic equations and quadratic functions (only proportional relationship to squares of a quantity), we paid particular attention to the following.

(2) Instructional considerations

① Understanding the meaning of letters

The letter a is used as a proportional constant in Grade 7. However, even though the phrase "proportional constant" is being used, when students must determine the value of a from the graph of the proportional relationship, it is being considered as an unknown. Moreover, while examining how the change in the proportional constant influences the graph of the proportional relationship, it is being used as a variable. On the other hand, when it is used in an identify such as an equation showing the distributive property of multiplication, it is used as a generalized number. Although the same letter, a, is being used, its meaning changes depending on the context. Therefore, it is important for the teacher to be aware that there are multiple meaning of letters and help students understand these meanings in a variety of situations.

Moreover, in elementary schools, often times the expression on the right side of an equation is the answer of calculations and the left side is the calculation problem. However, in the study of algebraic expressions, students need to look at an algebraic expression such as $3(a + b)$ shows both the process of calculation and the result of the calculation (Process-Product-Dilemma). Thus, to promote students' understanding, we need to engage them in rich activities that require students to interpret algebraic expressions and equations, and encourage them to think carefully about whether they are being used for representing the calculation processes or the results of calculations.

② Application of letters in problem solving

We want students to be able to deepen their exploration of relationships among quantities by representing the relationships in algebraic equations with letters and clarifying the structures of the relationship. For example, the sums of a 2-digit number and the second 2-digit number obtained by reversing the digits in the original 2-digit number are always multiples of 11. We can extend this relationship further without necessarily proving it. For example, we can focus on the number of digits, and try to examine what happens if we had a 3-digit number. Or, we can focus on the operation, and investigate what happens if we found the difference between those 2-digit numbers instead of the sum.

	2-digit #	3-digit #	4-digit #	5-digit #
Subtraction	Multiples of 9	Multiples of 99	~~Multiples of 999~~	- - -
Addition	Multiples of 11	~~Multiples of 111~~	- - -	

If we summarize the extensions in a table, it is as shown in the table above. Thus, we can see that it is possible to extend the original observation to the subtraction, and to the difference of two 3-digit numbers. However, the pattern cannot be extended any further. On the other hand, reflecting on a proof and investigate the conditions that produce these patterns based on the proof, we could discover the essence of this problem, and we can not only figure out the cases for 3- and 4-digit situations, but also we could understand the meaning of "switching digits" in the 2-digit number. We could say that this is also an example of reasoning with functions. We have been providing these opportunities across grade levels. Students need to be fluent in manipulating algebraic expressions appropriately.

Therefore, we believe our instruction should not just focus on developing skills to manipulating algebraic expressions or formal study of functions based on mathematical definition. Rather, we want to teach our students in such a manner that they can think about and understand the merits of reasoning based on experiences.

4 About students

Students have learned about calculations with algebraic equations, simultaneous linear equations, and linear functions in previous grades. In their study, they engaged in activities to investigate the nature of changes using algebraic expressions and equations, generalizing observed patterns, representing the patterns as functions and determining unknown numbers. During lessons, we encouraged them to be conscious of their own problem solving processes and reflect on "things that need to be reasoned," and "perspective for reasoning." This year, they have learned about factoring, various patterns of numbers, and quadratic equations. We tried to teach these ideas so that students can sense mathematical necessity.

Students do not necessarily feel that they are weak in mathematics, but there are some students who find it difficult to explain their own ideas. However, students have had experiences of thinking about and resolving challenges other students felt, and the classroom atmosphere is such that a student can openly share things they do not understand. Many students are willing to share their own ideas, and simple ideas often arise from the class.

5 Unit plan

Section 1 Introduction: Graphs of quadratic functions (3 lessons)
Section 2 Expressions of sums and graphs of quadratic functions (5 lessons)
Section 3 Expressions of products and quadratic equations (6 lessons)
Section 4 Need for square roots and quadratic equations (6 lessons)
Section 5 Application of expressions of sums and products to proofs (3 lessons)
(Today's lesson is Lesson 1 of 3)
Section 6 Quadratic equations and quadratic functions (10 lessons)

6 Today's lesson

(1) Goals of the lesson

Through the examination of a calculation technique, students will understand the merits of expressions of sums and products and the base-10 numeration systems. Students can investigate the essence of the calculation technique and try to extend the idea.

(2) About mathematics in today's lesson

Today's lesson will open by sharing some of the following calculations.

15 × 15 = 225
25 × 25 = 625
35 × 35 = 1225
45 × 45 = 2025
55 × 55 = 3025
65 × 65 = 4225

What are common in these calculations are:

- These calculations are all squaring of numbers whose ones digit is 5
- The last 2 digits in the products are all 25
- The digits in the hundreds place and above may be calculated by (digit in the tens place) × (digits in the tens place + 1)

If we simply consider these calculations as squaring of 2-digit number, students might try to prove this pattern by using algebraic equations,

$$(10a + b)^2 = 100a^2 + 20ab + b^2.$$

But, this equation does not really explain the patterns we observed.

If we represent the original number as $10a + 5$, then square, we get

$$(10a + 5)^2 = 100a^2 + 100a + 25.$$

But, this alone cannot explain the patterns either. If a student simply focuses on calculations, he or she might try to factor out 25 or other meaningless calculations.

This problem requires students to constantly reflecting on the original observations and identify the conditions necessary for the pattern. Therefore, this is an ideal problem for Grade 9 students who have learned about equivalent algebraic expressions through the study of factoring and expanding products of algebraic expressions.

The original calculations are special instances of the fact that the product of $10a + b$ and $10a + c$ can be expressed as $100a(a + 1) + bc$, when $b + c = 10$. The calculations will be as shown below:

$$(10a+b)(10a+c)$$
$$=\ 100a^2+10a(b+c)+\ bc$$
$$= 100a^2+100a+bc$$
$$= 100a(a+1)+\ bc$$

(3) Flow of the lesson

Time	Learning activity	Anticipated responses	※ Points of consideration 〇 Assessment
5 min.	[Introduction] • Write on the board: 15 × 15 = 225 45 × 45 = 2025 35 × 35 = 1225 • I have not memorized these. I'm just calculating really fast.	• You are really fast! • Did you memorize them?	
7 min.	[Step 1] Let's find the rule for the quick calculation. Independent problem solving (1)	• Calculate others 55 × 55 = 3025 95 × 95 = 9025 • Organize 15 × 15 = 225 25 × 25 = 625 35 × 35 = 1225 45 × 45 = 2025 55 × 55 = 3025 • Use algebraic expressions $(10a+5)^2$ $=100a^2+100a+\ 25$ (Inductively reason) 5 × 5 = 0 25 15 × 15 = 2 25 25 × 25 = 6 25 35 × 35 = 12 25 45 × 45 = 20 25 55 × 55 = 30 25 ○ The last 2 digits are 25. ○ The numbers formed by the digits in the hundreds place and above are increasing by 2, 4, 6, 8, 10, …	※ Make sure students understand the merits of organizing for discovering patterns. ※ If there is any student who is considering 5 x 5 as the case for the tens digit being a 0, make sure to discuss that idea. (This idea might be more common among those students who take an inductive approach.)

		(Reason to generalize) 5 × 5 = 0 25 15 × 15 = 2 25 25 × 25 = 6 25 35 × 35 = 12 25 45 × 45 = 20 25 55 × 55 = 30 25 ○ The last 2 digits are 25. ○ The numbers formed by the digits in the hundreds place and above are (tens digit) × (tens digit + 1)	
5 min.	Sharing of ideas from independent problem solving (1)		※ Share how they considered the numbers on the left hand side of the equation. ※ It is possible that students discover 2 different patterns (from the inductive reasoning and generalization). However, because the proof for the latter is easier, if no student comes identify the pattern, ask, “I wonder if we need to start calculating with 1 in order.” (It is hoped that students will seek for the generalization pattern because the original calculations were not presented in order.)
8 min.	[Step 2]		
	Let’s prove that when we square a 2-digit number ending in a 5, the last 2-digit of the product is 25 and the number formed by the digits in the hundreds place and above will be (tens digit) × (tens digit + 1).		
	Independent problem solving (2)	• Cannot go beyond $(10a+5)^2 = 100a^2 + 100a + 25$. • Can go as far as $(10a+5)^2 = 100a^2 + 100a + 25 = 25(4a^2 + 4a + 1) = 25(2a+1)^2$.	※ The task will be set up based on the problems students develop in independent problem solving (1).

		• Cannot figure out the conditions for *b* and stop after calculating $(10a+b)^2 = 100a^2 + 20ab + b^2$. • $(10a+5)^2$ $= 100a^2 + 100a + 25$ $= 100(a^2 + a) + 25$ $= 100a(a+1) + 25$. • Calculate $(10a+b)^2 = 100a^2 + 20ab + b^2$, then continue the calculation by substituting *b* = 5: $100a^2 + 20a \times 5 + 25$ $= 100a^2 + 100a + 25$ $= 100(a^2 + a) + 25$ $= 100a(a+1) + 25$.	※ When there are many students who used the method on the left, during the sharing ideas from independent problem solving (2), have students compare and contrast different solutions so that students might think about the condition for *b*.
10 min.			※ Encourage students who were stuck to share. (If students do not volunteer, the teacher will share the idea and ask if anyone who had a similar problem.) Develop a shared understanding of the challenge. Ideally, we want students to verbalize, “I don’t know how to proceed with the calculation” or “I don’t know what to do next.” ※ As students share in response to the above, ask them “what their goal was.” 〇 Students will understand the merits of expressions of sums and products. ※ Make sure students understand the meaning of factoring out 100. 〇 Students will understand the merits of the base-10 numeration systems.

15 min.	[Summary] • What do we need to pay attention to when we are manipulating algebraic expressions? • What if the ones digit isn't 5?	• We need to have a clear goal for manipulation. • We need to manipulate algebraic expressions with an understanding if the expression is an expression of sums or products. • If the ones digit is not 5, for example 23 × 23 = 529, there is no pattern. • If the ones digit is not 5, the term $20ab$ gets in the way to have a simple technique for calculation.	※ Students have learned that factoring and expanding products of polynomials are ways to manipulate algebraic expressions. Therefore, we will try to make sure students understand what the goal of manipulations is. ※ After students understand that we needed the term $100a$, depending on the availability of time, the teacher will suggest ideas like: • if $b + c = 10$, then for example, 23 × 27 = 621 and 34 × 36 = 1 • $(10a + b)(10a + c)$ $= 100a^2 + 10a(b + c) + bc$ $= 100a^2 + 100a + bc$ $= 100a(a + 1) + bc$ 〇 Students can investigate the essence of the calculation technique and try to extend the idea.