## 6. The Unit Plan and Evaluation Plan (11 lessons)

## Basic Course

|  | Basic Course |  |  |
| :---: | :---: | :---: | :---: |
| Lesson | Goals | Learning Activities | Evaluation Method |
|  | (1) Division of Fractions |  |  |
| 1 | O Students understand the meaning of dividing by fraction and are able to explain why the math sentence is a fraction : fraction sentence. | - Students solve a problem related to the quantity of paint and area of board that the paint will be used to cover (in the order of whole number to fraction) <br> - Student solve a problem whose math sentence becomes a fraction $\div$ fraction math sentence. <br> - Students explain why the divisor is a fraction. (Display number line and math sentence with words using projector) | A --- (1) [speaking/presentation, notebook] <br> B --- (1) [Speaking/presentation, notebook] |
| 2 | O Students think about how to calculate a fraction $\div$ fraction math sentence. | - Students notice that they don't know how to calculate division when the divisor is a fraction. They will show foresight about how to change the divisor into a whole number, so they can use their prior knowledge (i.e., knowledge about solving fraction $\div$ whole number). <br> - Students think about how to calculate fraction $\div$ fraction by using the property of division that the teacher underscores and/or shows. <br> - Students present their ideas. <br> - Students identify the commonality of the methods. <br> - Students generalize the calculation method for fraction $\div$ fraction problems using the commonality of different methods and verbally summarizes the process of calculation using words. | B --- (2) [Speaking/presentation, notebook] |
| 3 | O Students understand that calculation can be done easily if fractions are simplified (reduced) in the process of calculation. O Students understand how to calculate whole number $\div$ fraction and division involving mixed numbers. Students are able to | - Students review the calculation process of fraction $\div$ fraction by solving a calculation problem. <br> - Students solve $\frac{9}{14} \div \frac{3}{4}$. <br> - Students learn that they could simplify (reduce) fractions in the process of calculation. Students think about common factors of 14 and 4, and 9 and 3 with the teacher. <br> - Students notice that if they simplify (reduce) fractions in the process of calculating, the calculation becomes easier. | A --- (2) <br> [Speaking/presentation, notebook. <br> C --- (2) [Notebook] <br> C --- (3) [Notebook] |


|  | complete the calculation. | - Students solve $4 \div \frac{9}{2}$. <br> - Students solve $\frac{2}{3} \div 3 \frac{1}{5}$. <br> - Students solve three application problems. | D --- (2) [Speaking/presentation, notebook] |
| :---: | :---: | :---: | :---: |
| 4 | O Students understand that when the divisor in division problems is a proper fraction, the quotient becomes greater than the dividend. <br> O Students understand how to calculate three fractions using a combination of multiplication and division. Students are able to successfully complete the calculation. | - Students pay attention to the quotients when a divisor is divided by either a proper fraction or a mixed number. <br> - Students grasp the relationship of quantities visually by studying the number line that teacher shows. <br> - Students understand that when the divisor is a proper fraction, the quotient becomes greater than the dividend. <br> - Students think about how to calculate $\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5}$. <br> - Students solve three application problems. | B --- (2) [Speaking/presentation, notebook] <br> C --- (4) [Speaking/presentation, notebook] |
| 5 | O Students understand that the multiplication/division calculations that include a combination of fractions, decimals, and whole numbers can be easily calculated by converting all the numbers to fractions. Students are able to successfully complete the calculation. | - Students think about how to calculate $0.3 \div \frac{3}{5}$. <br> - Students present ideas and learn that the decimal can be converted to fractions. <br> - Students notice that some fractions cannot be converted to decimals, therefore, these calculations are easier if decimals and whole numbers are converted to fractions. <br> - Students solve two application problems. | B --- (3) <br> [Speaking/presentation, notebook] C --- (5) [notebook] |
| 6 | O Students develop deeper understanding of the process for making decisions about choosing appropriate operations using a number line. | - Students think about a math sentence that matches/represents the problem which asks students to find the weight of 1 m , using the number line the teacher provides. <br> - Students think of a math sentence that matches/represents the problem about finding the length for 1 kg , using the number line the teacher provides. <br> - Students think about a continuation to the word problem and write a math sentence. | B --- (4) [Speaking/presentation, notebook] |

## (2) "Times as Much" with Fractions and Multiplication/Division of Fractions

|  | (2) "Times as Much" with Fractions and Multiplication/Division of Fractions |  |  |
| :---: | :---: | :---: | :---: |
| 1 | O Students understand that the value that represents "times as much" can be found by division, even if both the comparing quantity and the base quantity are fractions. | - Students think about how many times $\frac{5}{4} \mathrm{~m}$ is as much as $\frac{1}{2} \mathrm{~m}$. <br> - Students identify the divisor and the dividend by thinking about how the same situation would be with whole numbers. <br> - Students think about how many times $\frac{3}{8} \mathrm{~m}$ is as much as $\frac{1}{2} \mathrm{~m}$. <br> - Students summarize the concept of "times as much" by expressing the quantities on a number line. | C --- (6) <br> [Speaking/presentation, notebook] |
| 2 | O Students understand that the comparing quantity can be found using: (base quantity) $\times$ (times as much) $=$ (comparing quantity). | - Students think about how to find the cost of something that is $\frac{6}{5}$ times as much as $¥ 600$ and $\frac{3}{5}$ times as much as $¥ 600$. <br> - Students show the relationship on a number line and students estimate the cost using it. <br> - Students understand that the cost can be found by using multiplication. <br> - Work on the problem in the textbook, fill in the word in the $\square$, and think about the meaning of the math sentence. | C --- (7) [Speaking/presentation, notebook] |
| 3 | O Students understand that the base quantity can be found using: (comparing quantity) $\div$ (times as much) $=$ (base quantity). | - Students write quantities on a number line that represents $¥ 900$ is $\frac{5}{3}$ times as much as the original price. <br> - Students assign x to the cost and express the math sentence that represents the original price. <br> - Students solve 1 appropriate application problem. | C --- (8) [Speaking/presentations, notebook] |
|  | (3) Summary |  |  |
| 1 | Students solve problems by applying what they learned in the unit. | - Students solve problems in Power Builder. | C --- (1) to (8) [Notebook] |
| 2 | O Students check their understanding of the content of the unit and solidify their learning. | - Students solve problems in Mastery Problems. | D --- (4) [Notebook] |

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Regular Course

|  | Regular Course |  |  |
| :---: | :---: | :---: | :---: |
| Lesson | Goals | Learning Activities | Evaluation Method |
|  | （1）Division of Fractions |  |  |
| 1 | O Students understand the meaning of dividing by fractions and are able to explain why the math sentence is fraction $\div$ fraction． | －Students solve a problem related to the quantity of paint and area of board that the paint can cover．（in the order of whole number to fraction） <br> －Student solve a problem whose math sentence becomes fraction $\div$ fraction． <br> －Students explain why the divisor is a fraction．（Display number line and math sentence in words） | A－－－（1）［speaking／presentation， notebook］ <br> B－－－（1）［Speaking／presentation， notebook］ |
| 2 | O Students think about and explain how to calculate a fraction $\div$ fraction problem． | －Students notice that they don＇t know how to calculate division when the divisor is a fraction．They show foresight about changing the divisor into a whole number so they can use their prior knowledge（fraction $\div$ whole number）． <br> －Students think about how to calculate fraction $\div$ fraction by using the property of division that the teacher shows． <br> －Students present their ideas． <br> －Students identify the commonality of the methods． <br> －Students generalize the calculation method of fraction $\div$ fraction using the commonality of different methods and verbally summarize the process of calculation using words． | B－－－（2）［Speaking／presentation， notebook］ |
| 3 | O Students understand that calculations can be easier if fractions are simplified（reduced）in the process of calculation． <br> O Students understand how to calculate whole number $\div$ fraction and division that involves mixed numbers．Students able to successfully complete the calculation． | －Students review the calculation process of fraction $\div$ fraction by solving a calculation problem． <br> －Students solve $\frac{9}{14} \div \frac{3}{4}$ ． <br> －Students present two solution methods：（1）simplify（reducing）fractions in the process of calculation and（2）no reduction of fractions until the answer is found． <br> －Students compare the two methods mentioned above，and notice that it is easier to calculate if they simplify fractions in the process of calculating． <br> －Students solve $4 \div \frac{9}{2}$ ． <br> －Students solve $\frac{2}{3} \div 3 \frac{1}{5}$ ． <br> －Students solve six application problems． | A－－－（2） <br> ［Speaking／presentation，notebook． <br> C－－－（2）［Notebook］ <br> C－－－（3）［Notebook］ <br> D－－－（2）［Speaking／presentation， notebook］ |


| 4 | O Students understand that when the divisor in division is a proper fraction，the quotient will be greater than the dividend． <br> O Students understand how to calculate three fractions with a combination of multiplication and division．Students are able to do the calculation． | －Students pay attention to the quotients when a divisor is divided by either a proper fraction or a mixed number． <br> －Students grasp the relationship of quantities visually by studying the number line the teacher shows． <br> －Students understand that when the divisor is a proper fraction，the quotient becomes greater than the dividend． <br> －Students think about how to calculate $\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5}$ ． <br> －Students solve three application problems． | B－－－（2）［Speaking／presentation， notebook］ <br> C－－－（4）［Speaking／presentation， notebook］ |
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| 5 | O Students understand that the multiplication／division calculations that include a mix of fractions， decimals，and whole numbers can be easily calculated by converting all the numbers to fractions．Students are able to do this calculation． | －Students think about how to calculate $0.3 \div \frac{3}{5}$ ． <br> －Students present ideas and learn that there are two ways to do the calculations． <br> －Students solve the problem $0.3 \div \frac{3}{5} \times 2$ ． <br> －Students notice that some fractions cannot be converted to decimals；therefore， it is easier to calculate if both decimals and whole numbers are converted to fractions． <br> －Students solve three application problems． | B --- (3) <br> ［Speaking／presentation，notebook］ C－－－（5）［notebook］ |
| 6 | O Students develop deeper understanding of the process for making decisions to choose appropriate operations using a number line． | －Students think about a math sentence that matches／represents the problem about finding the weight for 1 m ；they use a number line the teacher provides． <br> －Students think about a math sentence that matches／represents the problem that is about finding the length for 1 kg ；their thinking is aided by drawing a number line without teacher or classmate help． <br> －Students think about a continuation of the word problem and write a math sentence． | B－－－（4）［Speaking／presentation， notebook］ |
|  | （2）＂Times as Much＂with Fractions and Multiplication／Division of Fractions |  |  |
| 1 | O Students understand that a value that shows＂times as much＂can be found by division，even when the comparing quantity and the base quantity are fractions． | －Students think about how many times $\frac{5}{4} m$ is as much as $\frac{1}{2} m$ and how many times $\frac{3}{8} \mathrm{~m}$ is as much as $\frac{1}{2} \mathrm{~m}$ ． <br> －Students identify the divisor and the dividend by thinking about the same situation with whole numbers． <br> －Students summarize the concept of＂times as much＂by expressing the quantities on a number line． | C --- (6) <br> ［Speaking／presentation，notebook］ |
| 2 | O Students understand that the comparing quantity can be found using：（base quantity）x（times as much $)$＝comparing quantity． | －Students think about how to find the cost of something that is $\frac{6}{5}$ times as much as $¥ 600$ and $\frac{3}{5}$ times as much as $¥ 600$ ． <br> －Students and teacher together construct and use a number line to estimate the | C－－－（7）［Speaking／presentations， notebook］ |


|  |  | cost. <br> - Students understand that the cost can be found by using multiplication; they find the cost by making calculations. <br> - Work on the problem in the textbook, fill the word in the $\square$, and think about the meaning of the math sentence. |  |
| :---: | :---: | :---: | :---: |
| 3 | O Students understand that the base quantity can be found using: (comparing quantity) $\div$ (times as much) = base quantity. | - Students and teacher together construct a number line that represents $¥ 900$ is $\frac{5}{3}$ times as much as the original price. <br> - Students assign x to the cost and express the math sentence that represents the original price. <br> - Students solve 1 appropriate application problem. | C --- (8) [Speaking/presentation, notebook] |
|  | (3) Summary |  |  |
| 1 | O Students solve problems by applying what they learned in the unit. | - Students solve problems in Power Builder. | C --- (1) to (8) [Notebook] |
| 2 | O Students check their understanding of the content of the unit and solidify their learning. | - Students solve problems in Mastery Problems. | D --- (4) [Notebook] |

Advanced Course

|  | Advanced Course |  |  |
| :---: | :---: | :---: | :---: |
| Lesson | Goals | Learning Activities | Evaluation Method |
|  | (1) Division of Fractions |  |  |
| 1 | O Students understand the meaning of dividing by a fraction and are able to explain why the math sentence is a fraction $\div$ fraction sentence. | - Students solve a story problem that requires them to establish that the math lesson and problem is about fraction division, fraction $\div$ fraction. <br> - Student think about and explain why the divisor becomes a fraction (use number line, word math sentence) <br> - Students think about how to calculate fraction $\div$ fraction problems. | A --- (1) [Speaking/presentation, notebook] <br> B --- (1) [Speaking/presentation, notebook] |
| 2 | O Students think about and explain how to calculate fraction $\div$ fraction. Moreover, they understand the presented ideas, find the commonality among them, and generalize the idea to construct the generalized formula for the calculation. | - Students present their ideas about how to calculate fraction $\div$ fraction. (process the math sentence, area model, etc.) <br> - Students deepen their understanding of other students' ideas. <br> - Students discuss and compare the presented ideas. Then, they find the differences and commonality of the ideas. <br> - Students generalize the calculation method of fraction $\div$ fraction using the commonality of the ideas; they summarize the process of calculation using words and a math sentence with letters and symbols. | B --- (2) [Speaking/presentation, notebook] |
| 3 | O Students understand that calculation can be made easier if fractions are simplified in the process of calculation. <br> O Students understand how to calculate whole number $\div$ fraction and division involving mixed numbers. Students able to successfully complete the calculation. | - Students review the calculation process of fraction $\div$ fraction by solving a calculation problem. <br> - Students solve $\frac{9}{14} \div \frac{3}{4}$. <br> - Students present two solution methods: (1) simplify (reducing) fractions in the process of calculation and (2) no reduction of fractions until the answer is found. <br> - Students compare the two methods mentioned above, and notice that it is easier to calculate if they simplify fractions in the process of calculation. <br> - Students solve $4 \div \frac{9}{2}$. <br> - Students solve $\frac{2}{3} \div 3 \frac{1}{5}$. <br> - Students solve eleven application problems. | A --- (2) <br> [Speaking/presentation, notebook. <br> C --- (2) [Notebook] <br> C --- (3) [Notebook] <br> D --- (2) [Speaking/presentation, notebook] |


| 4 | O Students understand that when the divisor of a division problem is a proper fraction，the quotient becomes greater than the dividend． O Students understand how to calculate three fractions using a combination of multiplication and division．Students are able to do the calculation． | －Students solve two problems and compare the math sentences and answers． <br> －Students draw number lines and grasp the relationships of quantities visually． <br> －Students understand that when the divisor is a proper fraction，the quotient becomes greater than the dividend． <br> －Students think about how to calculate $\frac{3}{4} \div \frac{6}{5} \times \frac{1}{5}$ ． <br> －Students solve seven application problems． | B－－－（2）［Speaking／presentation， notebook］ <br> C－－－（4）［Speaking／presentation， notebook］ |
| :---: | :---: | :---: | :---: |
| 5 | O Students understand that the multiplication／division calculations that are a mixture with fractions， decimals，and whole numbers can be easily done by converting all the numbers to fractions．Students are able to do the calculation． | －Students think about how to calculate $0.3 \div \frac{3}{5}$ ． <br> －Students present ideas and learn that the decimal can be converted to a fraction． <br> －Students notice that some fractions cannot be converted to decimals；but decimals and whole numbers can be converted to fractions to carry out the calculations． <br> －Students solve two application problems． | $\begin{aligned} & \text { B --- (3) } \\ & \text { [Speaking/presentation, notebook] } \\ & \text { C --- (5) [notebook] } \end{aligned}$ |
| 6 | O Students develop a deeper understanding for making decisions about choosing appropriate operations． | －Students think about a math sentence that matches／represents the problem that asks about finding the weight for 1 m ，and the problem that asks about finding the length of 1 kg （by drawing a number line）． <br> －Students think about a continuation of the word problem and write a math sentence． <br> －Students create story problems and present their problem（s）to other students to solve． | B－－－（4）［Speaking／presentation， notebook］ |
|  | （2）＂Times as Much＂with Fractions and Multiplication／Division of Fractions |  |  |
| 1 | O Students understand that the value that shows＂times as much＂can be found by division，even if the comparing quantity and the base quantity are fractions． | －Students think about how many times $\frac{5}{4} \mathrm{~m}$ is as much as $\frac{1}{2} \mathrm{~m}$ and how many times $\frac{3}{8}$ m is as much as $\frac{1}{2} \mathrm{~m}$ ． <br> －Students identify the relationship among the quantities visually by drawing a number line． <br> －Students understand how to find＂times as much．＂ <br> －Students solve two application problems． | $\begin{aligned} & \text { C --- (6) } \\ & \text { [Speaking/presentation, notebook] } \end{aligned}$ |
| 2 | O Students understand that the comparing quantity can be found using：（base quantity）$\times$（times as much）＝comparing quantity． | －Students think about how to find the cost of something that is $\frac{6}{5}$ times as much as $¥ 600$ and $\frac{3}{5}$ times as much as $¥ 600$ ． <br> －Students estimate the cost using a number line． | C－－－（7）［Speaking／presentation， notebook］ |


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|  |  | - Students understand that that the cost can be found by multiplication. Students find the cost by calculation. <br> - Students confirm the math sentence that helps them find the price of the pencil sharpener, and describe the math sentence using words. Students verbally describe how to find the prices of a colored pencil set and a notebook using words. |  |
| :---: | :---: | :---: | :---: |
| 3 | O Students understand that the base quantity can be found using: (comparing quantity) $\div$ (times as much) = base quantity. | - Students draw a number line to represent the relationship of quantities in a problem: $¥ 900$ is $\frac{5}{4}$ times as much as the original price. <br> - Students assign $x$ to the cost and express the math sentence that shows the original price. <br> - Students find the original prices (finding the base quantity). <br> - Students solve 1 appropriate application problem. | C --- (8) [Speaking/presentation, notebook] |
|  | (3) Summary |  |  |
| 1 | O Students solve problems by applying what they learned in the unit. | - Students solve problems in Power Builder. They also solve additional problems. | C --- (1) to (8) [Notebook] |
| 2 | O Students check their understanding of the content of the unit and solidify their learning. | - Students solve problems in Mastery Problems. | D --- (4) [Notebook] |

