2012 Grade 9 Mathematics Set B

The English translation is prepared by the Project IMPULS at Tokyo Gakugei University, Tokyo, Japan. (http://www.impuls-tgu.org/)

[1] The table below shows information about the International Space Station (ISS) and the weather satellite, Himawari 7.

(Photo of International Space Station (ISS))

International Space Station (ISS)

(Photo of weather satellite, Himawari 7)

Weather satellite, Himawari 7

	ISS	Himawari 7
Total length	About 108.5 m × About 72.8 m (approximately the size of a soccer field)	About 30 m
Height from the surface of the Earth (altitude)	About 400 km	About 35800 km
Time it takes to go around the Earth	About 1.5 hours	About 24 hours

Answer the following questions (1) and (2).

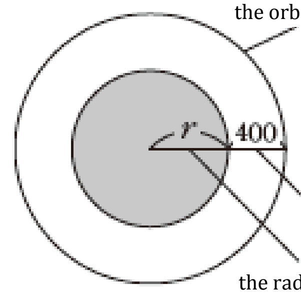
(1) We are going to think about the relative positions of the Earth, ISS, and Himawari 7 using a globe as the Earth. If the ISS is traveling at the height of about 1 cm from the surface of the globe, what will be the approximate height of the path of Himawari 7 from the surface of the globe? Select the correct answer from (a) through (e) below.

(a) About 9 cm (b) About 16 cm (c) About 36 cm

(d) About 90 cm (e) About 400 cm

(2) The path a satellite takes to go around the Earth is called its orbit. The difference of the lengths of the orbits of the ISS and of Himawari 7 can be determined as follows.

As shown on the right, if we consider the earth as a sphere of the radius of r km and the orbit of a satellite as a circle, we can express the radius of the orbit for the ISS as (r + 400) km and the lengths of the orbit to be 2π (r + 400) km.



Similarly, we can determine the length of the orbit for Himawari 7. Then, the difference between these 2 satellites can be calculated as shown below.

$$
\begin{aligned}
& 2\pi(r+35800) - 2\pi(r+400) \\
&= \cancel{2\pi r} + 2\pi \times 35800 - \cancel{2\pi r} - 2\pi \times 400 \\
&= 2\pi \times 35800 - 2\pi \times 400 \\
&= 2\pi \times (35800 - 400) \\
&= 2\pi \times 35400 \\
&= 70800\pi
\end{aligned}
$$

In this way, we know the difference in the lengths of the orbits of the two satellites is about 70800π km.

From what is written in [dashed box] we can determine something else about the difference in the lengths of the orbits. Select the correct one from (a) and (b) below. Then, explain why your choice is the correct statement.

(a) The difference in the lengths of the orbits is determined by the radius of the Earth.

(b) The difference in the lengths of the orbits does not depend on the radius of the Earth.

[2] Tomoya is investigating what he can say about the sums of three consecutive natural numbers.

If the numbers are 1, 2, and 3, then 1 + 2 + 3 = 6.

If the numbers are 2, 3, and 4, then 2 + 3 + 4 = 9.

If the numbers are 3, 4, and 5, then 3 + 4 + 5 = 12.

6 = 3 × 2

9 = 3 × 3

12 = 3 × 4

All 3 sums are multiples of 3, aren't they?

Based on his investigation above, he made the following prediction.

Tomoya's prediction

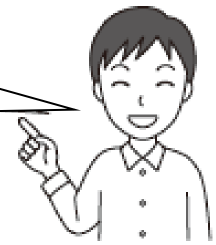
The sum of 3 consecutive natural numbers will be a multiple of 3.

If the numbers are 7, 8, and 9, then 7 + 8 + 9 = 24, and 24 = 3 × 8. So, as predicted, the sum is a multiple of 3 in this case, too.

Answer the following questions (1) and (2).

(1) We are going to explain why **Tomoya's prediction** will always be true. Complete the **explanation** below.

To explain why the sum is always a multiple of 3, we need to show that the sum is always a product of 3 and a natural number.



Explanation

Let n be the smallest of the three consecutive 3 natural numbers.

Then, the 3 consecutive natural numbers will be expressed as n, $n + 1$, and $n + 2$.

Therefore, the sum of 3 consecutive natural numbers is

$n + (n + 1) + (n + 2) =$

(2) Next, Tomoya wanted to investigate what would happen to the sums if "**3 consecutive natural numbers**" is changed to "3 consecutive even numbers." He calculated the sums for several cases.

If the numbers are 2, 4, and 6, then 2 + 4 + 6 = 12.

If the numbers are 8, 10, and 12, then 8 + 10 + 12 = 30.

If the numbers are 20, 22, and 24, then 20 + 22 + 24 = 66.

: :

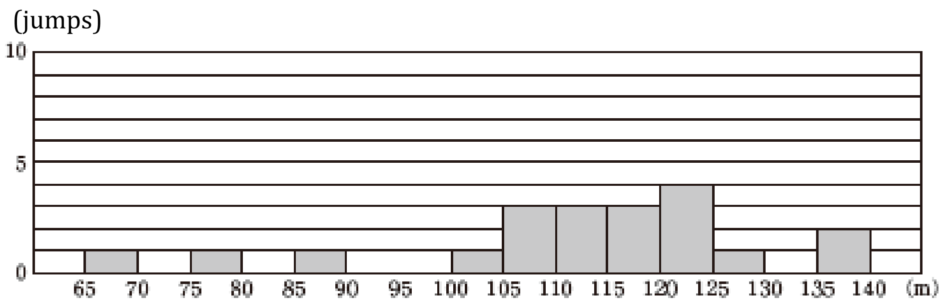
What prediction can you make about the sum of **3 consecutive even numbers**? Write your prediction in the format Tomoya used for **his prediction** on the previous page, " **__ will be a __**"

[3] Misaki was born in 1998. She became interested in the ski jump event in which the Japanese team won the gold medal during the Nagano Olympics held in 1998. In this competition, both the distance of a jump and the jumper's flying style are considered.

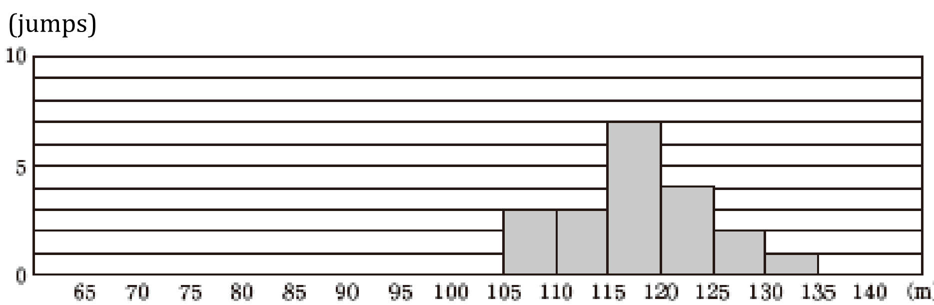
Misaki researched the jump distances of the two members of the Japanese team, Masahiko Harada and Kazuyoshi Funaki. The two histograms shown below summarize the jump distances of these two athletes from several International meets held during the 1998 season prior to the Nagano Olympics. From these histograms, we can tell, for example, that these two athletes made 3 jumps longer than or equal to 105 m but shorter than 110 m.

Photos of Masahiko Harada and Kazuyoshi Funaki.

Records for Harada



Records for Funaki



Answer the following questions (1) and (2).

(1) From the two histograms in the previous page, one can tell that these jumpers made the same number of jumps. What is the number of jumps each athlete made?

(2) Misaki wanted to think about who would have jumped farther had they each made one more jump based on the data shown in the histograms.

If you are going to choose one athlete who might make a longer jump based on your comparison of the two histograms and the characteristics you identified, which athlete would you pick? From (a) and (b) below, select one athlete. Then, explain why you chose that athlete **by comparing the characteristics of the histograms of these athletes**. You may justify a choice of either athlete.

(a) Harada

(b) Funaki

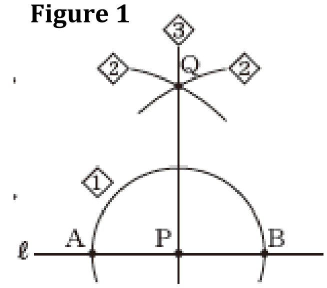
[4] We can construct the perpendicular line to line ℓ passing through point P on line *l* following steps 1, 2, and 3 as shown in **Figure 1**.

Step 1: Draw a circle centered at point P with an arbitrary radius.
Let the points of intersection between the circle and line *l* be points A and B.

Step 2: Draw circles centered at points A and B with equal radii so that the circles intersect each other. Let the point of intersection be point Q.

Step 3: Draw a line passing through points P and Q.

Figure 1



Answer the following questions (1) through (3).

(1) If we connect points Q, A, P, and B shown in Figure 1 in that order, we get △QAB. If we copy △QAB on a sheet of paper and fold it along line PQ, with which point will point A will match up? Write the name of that point.

(2) To show that line PQ in **Figure 1** is perpendicular to line *l*, we are going to prove PQ ⊥ *l*. From **Step 1**, we know that AP = BP. From **Step 2**, we know that QA = QB. Based on these observations, complete the following proof by showing that △QAP ≅ △QBP,..

Proof

With respect to △QAP and △QBP,

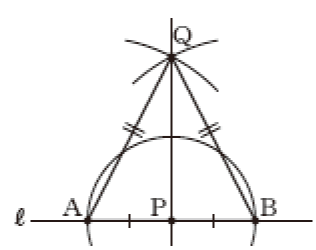
Since corresponding angles of congruent triangles are congruent,

∠APQ = ∠BPQ.

Since ∠APQ + ∠BPQ = ∠ APB = 180°,

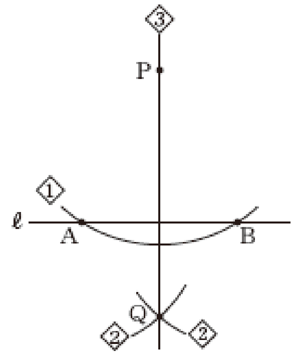
∠APQ = ∠BPQ = 90°.

Therefore, PQ ⊥ *l*.



(3) Even when point P is not on line *l*, we can construct a perpendicular line to line *l* following steps 1, 2, and 3 as described on the previous page. Figure 2 shows this construction.

Figure 2: Point P is not on line *l*



As shown in Figure 1 (on a previous page) and Figure 2, we can construct a line perpendicular to line *l* and passing through point P following the same **steps** whether or not point P is on line *l*. The reason we can use the same steps of construction is because the figure obtained by connecting points Q, A, P, and B in that order has a particular characteristic. That figure is described by one of the statements (a) through (d) below. Select the correct one.

(a) The figure is symmetric with line PQ as the line of symmetry.

(b) The figure is symmetric with line *l* as the line of symmetry.

(c) The figure is symmetric with point P as the center of symmetry.

(d) The figure is symmetric with the point of intersection between line *l* and line PQ as the center of symmetry.

[5] In the mathematics book, *Jinkoki*, published during the Edo period, there are many calculation techniques that are useful in everyday life. The figure below shows a method of determining the height of a tree.

From *Jinkoki* published in 1627

Shohta became interested in this idea, and he summarized the method of determining the height of a tree as follows.

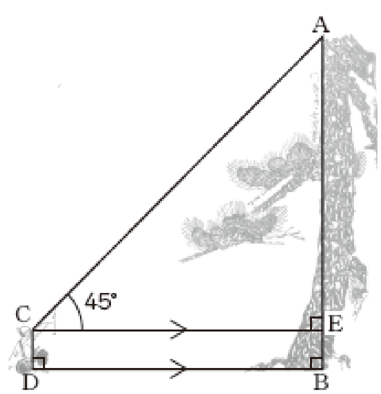
Method of determining the height of a tree

Steps

1 Let point A be the highest point of a tree and point B be the base of the tree. Move away from the tree so that the sight line to observe point A forms a 45° angle with a line parallel to the ground level.

2 Let point C be the person’s eye position and point D be where the person is standing. Measure the lengths of CD and DB.

3 The height of the tree, AB, is the sum of the lengths of CD and DB.



Key points

- Let point E be the point of intersection between AB and the line parallel to DB passing through point C. Since we cannot measure the length of AB directly, split AB into AE and EB. Then, measure each of those lengths by measuring the lengths of other segments that are equal to those two parts.
- If we assume both the tree and the person are perpendicular to the ground, then $AB \perp DB$, $CD \perp DB$, and $\angle AEC = 90°$.

Answer the following questions (1) through (3).

(1) If the height of the eye position, CD, is 1.2 *m*, and the length of DB is 8.3 *m*, determine the height of the tree using the method of determining the height of a tree described in the previous page.

(2) In step 2 of the method of determining the height of a tree, the reason we are measuring the lengths of CD and DB is because we can use the length of CD for EB and the length of DB for CE. The reason we can do so is because quadrilateral CDBE is a rectangle. Select the correct property of rectangles being used here from the statements (a) through (d) below.

(a) The four angles of a rectangle are all equal.

(b) The both pairs of opposite sides of a rectangle are parallel.

(c) The both pairs of opposite sides of a rectangle are equal in length.

(d) The lengths of the diagonals of a rectangle are equal.

(3) In the method of determining the height of a tree, instead of measuring the length of CE directly, the following method is used to determine the length of CE.

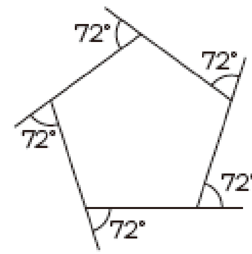
Using a property of rectangles, substitute the length of DB as the length of CE.

In determining the length of AE, in order to determine the length of AE indirectly, in step 1, $\angle$ACE in $\triangle$ACE is made to be 45°. Explain the method in a way similar to the explanation given in the box above.

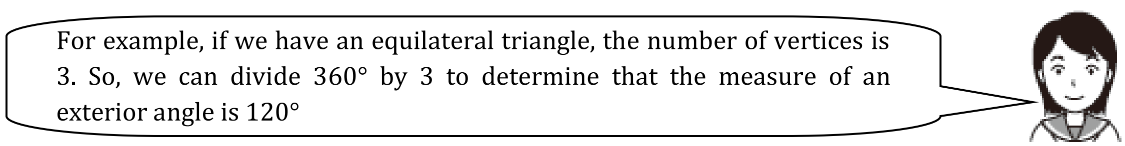
[6] Ryouta and Nanami are investigating the measures of an exterior angle of a regular pentagon based on the fact that the sum of the external angles of a polygon is 360°.

First, Ryouta determined the measure of an exterior angle of a regular polygon as follows.

> Since the measures of exterior angles of a regular polygon are equal, we can determine the measure of an exterior angle of a regular pentagon by dividing 360° by the number of vertices, 5.
>
> $360° \div 5 = 72°$
>
> Therefore, the measure of an exterior angle of a regular polygon is 72°.



Nanami realized that the measure of an exterior angle of a regular polygon other than a pentagon can be determined in the similar manner.



Answer the following questions (1) through (3).

(1) Determine the measure of an exterior angle of a regular dodecagon.

(2) We can tell that the following relationship exists: "If we fix the number of vertices in a regular polygon, the measure of an exterior angle is also fixed accordingly."

If we express this underlined relationship as shown below, write appropriate phrases to go into **[(1)]** and **[(2)]**.

[(1)] is a function of **[(2)]**.

(3) In order to investigate what type of function describes the relationship between the number of vertices of a regular polygon and the measure of its external angle, Ryouta and Nanami summarized what they figured out as shown below.

Summary

- The sum of exterior angles is 360° no matter what the number of vertices is.
- The measures of external angles are all equal.

From these facts, the measure of an external angle of a regular polygon can be determined by dividing the sum of external angles by the number of vertices.

Let x be the number of vertices in a regular polygon, and let y be the measure, in degrees, of an external angle. Then, what type of function describes the relationship between x and y in the summary above? Select the correct one from (a) through (c) below. In addition, explain why that function type is the correct one.

(a) proportional relationship

(b) inversely proportional relationship

(c) a linear function that is not proportional