2013 Grade 9 Mathematics Set B

[1] Yuko is thinking about suggesting to her father, who lacks physical activity in his daily routine, that he start exercise walking. Therefore, she researched exercise walking and summarized what she found as shown below.

Let's eliminate the lack of physical activity through exercise walking!

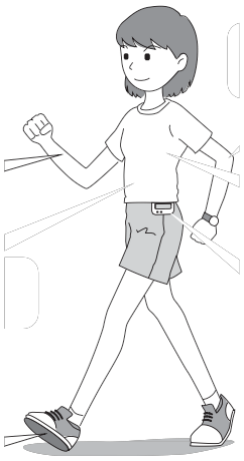
Determine the target heart rate and walk in a good form!

<u>Key points for proper walking form</u>

Swing your arms vigorously.

Tighten your stomach.

Land on your heels.



Bend your elbows at 90°.

Stick your chest out and straighten your back.

Wear a pedometer.

<u>How to determine a proper walking pace</u>

(1) Calculate the target heart rate during your exercise walk using the following formula.

(Target Heart Rate) = 88 – 0.4 × (Age) + 0.6 × (Heart Rate at Rest)

"Heart Rate at Rest" is determined by counting the number of heartbeats for 1 minute when you are at rest.

(2) During your walk, stop at a safe location and count your heartbeats for 1 minute. The key is not to exceed the "Target Heart Rate" during your walk.

Caution: Target Heart Rate is only a guideline. When you walk, consider your physical condition and mood on that day.

Answer the following questions (1) through (3).

(1) Yuko first decided to calculate her own Target Heart Rate. Yuko is 15 years old, and her Heart Rate at Rest was 80. Calculate Yuko's Target Heart Rate.

(2) Yuko's father and mother are both 45 years old. When they compared their Heart Rates at Rest one day, the difference was 10. Determine the difference in their Target Heart Rates.

(3) Yuko wanted to investigate how Target Heart Rate changes as a person's age increases. She assumed that the person's Heart Rate at Rest remains constant no matter how old he or she is.

When you make this assumption, Target Heart Rate will change along with the person's age. From (a) and (b) below, select the correct statement about the way Target Heart Rate changes. Also, explain why the selected statement is correct using the Target Heart Rate formula given on the previous page.

(a) As a person's age increases, Target Heart Rate will increase.

(b) As a person's age increases, Target Heart Rate will decrease.

[2] Daiki is investigating what kinds of numbers he will get when he finds the difference between a two-digit natural number and another two-digit number obtained by reversing the ones and tens digits in the original number.

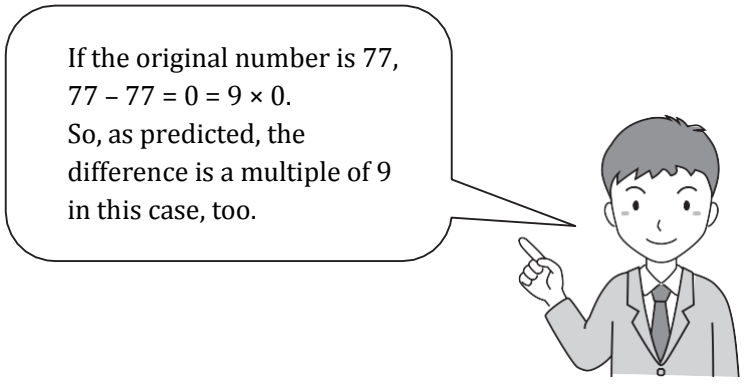
What he has checked:

When the original number is 41:	$41 - 14 = 27 = 9 \times 3$
When the original number is 53:	$53 - 35 = 18 = 9 \times 2$
When the original number is 28:	$28 - 82 = -54 = 9 \times (-6)$

Because the differences of the two numbers are multiples of 9 in the cases he checked above, he made the following prediction.

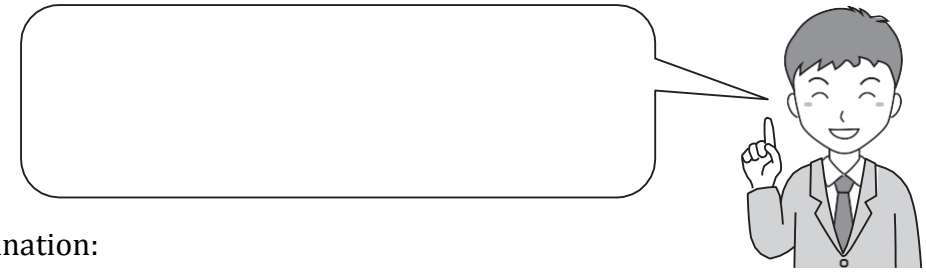
Prediction:

The difference between a two-digit natural number and the second two-digit number obtained by reversing the ones and tens digits in the original number will be a multiple of 9.



Answer the following questions (1) and (2).

(1) We are going to show that the prediction on the previous page is always true. Complete the explanation below.



Explanation:

If we call the tens digit in the original natural number as x and the ones digit as y, the two-digit natural number is, $10x + y$.

The two-digit number obtained by reversing the digits will be $10y + x$.

Therefore, their difference is,

$(10x + y) - (10y + x) =$

(2) Daiki then wanted to investigate the sum of a two-digit natural number and another two-digit number obtained by reversing the ones and tens digits in the original number. So, he checked a few cases.

When the original number is 21: $21 + 12 = 33$

When the original number is 35: $35 + 53 = 88$

When the original number is 48: $48 + 84 = 132$

⋮ ⋮

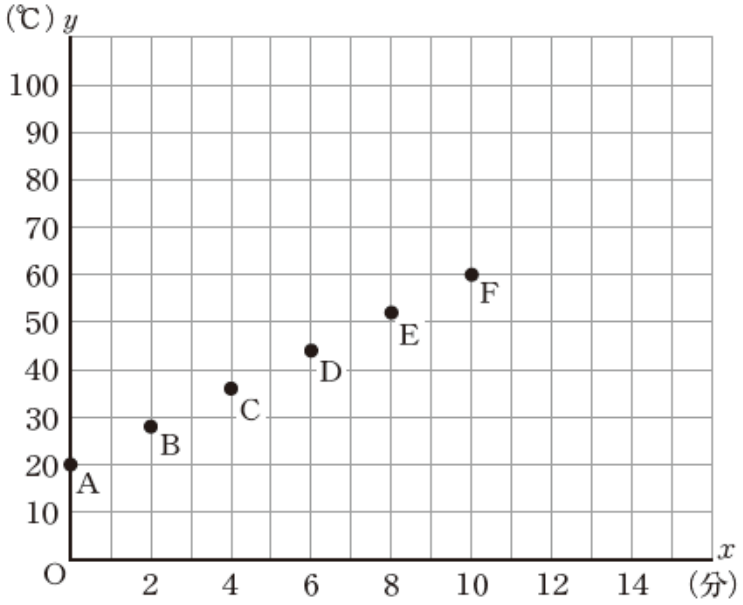
From these results, what prediction about the sum of a two-digit natural number and the number obtained by reversing its digits can we make? As in the prediction on the previous page, write your prediction in the form, "__ will be __."

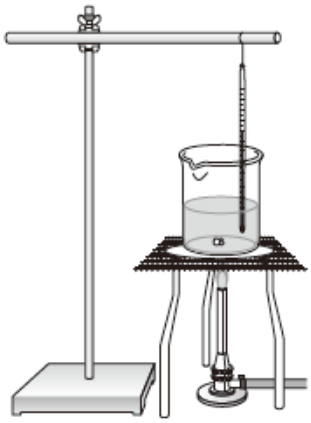
[3] Taichi investigated the change in water temperature as it was being heated. Then, he summarized in the table below the length of time water was heated and its temperature. He also drew a graph of the temperature, y °C, after water was heated for x minutes as shown below.

Results of Investigation

Length of time water was heated and its temperature

Length of time water was heated x (min.)	0	2	4	6	8	10
Temperature of water y °C	20.0	28.2	36.1	44.2	52.0	60.0





(min.)

Answer the following questions (1) through (3).

(1) How many °C did the water temperature rise in the first 10 minutes after Taichi started heating the water?

(2) In order to figure out the amount of time needed to raise the water temperature to 80 °C, Taichi decided to assume that points A through F, which show the relationship between the time and the temperature, all lie on the same line. Explain the method to find the amount of time needed to raise the water temperature to 80 °C. You do not have to actually calculate the amount of time.

(3) In (2), we investigated the temperature of water, y °C, at x minutes after Taichi began heating the water. As we did so, we checked the pairs of quantities, x and y, and assumed the points representing the pairs on the coordinates were on the same line.

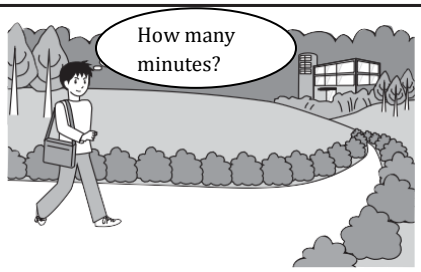
There is a situation in (a) through (d) below that can be considered in the same manner. Select the correct one.

(a) Altitude and temperature

What needs to be determined
The temperature at the 6th station on Mt. Fuji (altitude 2500 m) when the temperature at Kawaguchi Lake Weather Center at the base of Mt. Fuji (altitude 860 m) is 23.3 °C.

What is known
The temperature, y °C, at a certain location will decrease as the altitude, x m, increases at the rate of about 0.6 °C for every 100 m till about the altitude of 10,000 m.

(b) Speed and time



What needs to be determined
The amount of time it takes to get to the library that is located 2100 m from home,while walking at the speed of 70 m per minute.

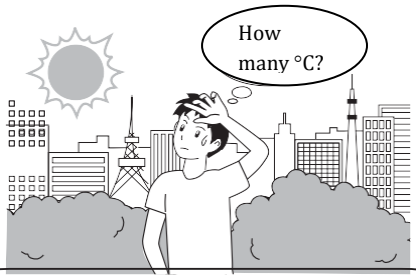
What is known
When you travel a distance of y m in x minutes, the product of x and y will be constant.

(c) Weight and cost

What needs to be determined
The cost of mailing a package that weighs 90 g.

What is known
The cost, y yen, for mailing a package weighing x g is set so that a package weighing up to 50 g will cost 120 yen, a package weighing up to 100g will cost 140 yen, and so on.

(d) Time and temperature



What needs to be determined
The temperature at 15 o'clock on the day when the temperature at the sun rise was 10 °C.

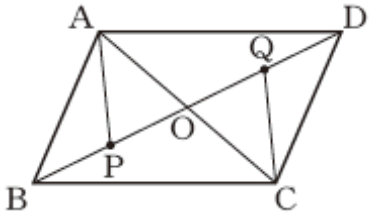
What is known
On a sunny day, the temperature, y °C, at x hours after the sun rise will continue to increase until about 14 o'clock and then will decrease until the sunrise of the following day.

[4] Yuto is thinking about the following problem.

Problem

Let the point of intersection of the diagonals of parallelogram ABCD be O, as shown on the right. Then, let points P and Q be points on segments OB and OD, respectively, such that BP = DQ.

Prove that AP = CQ.

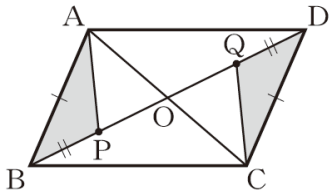


Answer the following questions (1) and (2).

(1) Yuto thought about the following Proof Plan 1. It is possible to prove that AP = CQ following Proof Plan 1.

Proof Plan 1

1. In order to prove that AP = CQ,
we need to show ΔABP ≅ ΔCDQ.
2. We need to find sides and angles in
ΔABP and ΔCDQ that are congruent to each other.

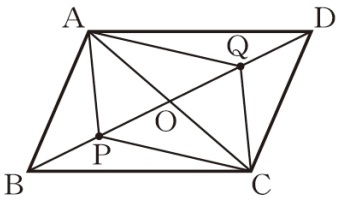


First, based on properties of parallelogram ABCD, we know AB = CD, and from the given condition we already know that BP = DQ.

3. By using (2), it seems possible to show that ΔABP ≅ ΔCDQ.

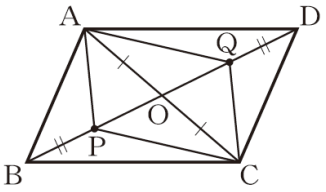
Prove that AP = CD by following Proof Plan 1.

(2) It is also possible to prove that AP = CQ by drawing segments AQ and CP, as described in Proof Plan 2 shown below.



Proof Plan 2

.1. In order to prove that AP = CQ, we need to show that quadrilateral APCQ is a parallelogram.



2. In quadrilateral APCQ, based on properties of parallelogram ABCD, we know that OA = OC.

3. By using (2) and the given condition, BP = DQ, it seems possible to show that quadrilateral APCQ is a parallelogram because [ ].

The statement that goes into the [ ] in Proof Plan 2 is one of the following statements (a) through (d). Select the correct statement.

(a) the diagonals intersect at their mid-points.

(b) the diagonals are perpendicular to each other.

(c) the lengths of the diagonals are equal.

(d) the diagonals are perpendicular and their lengths are equal.

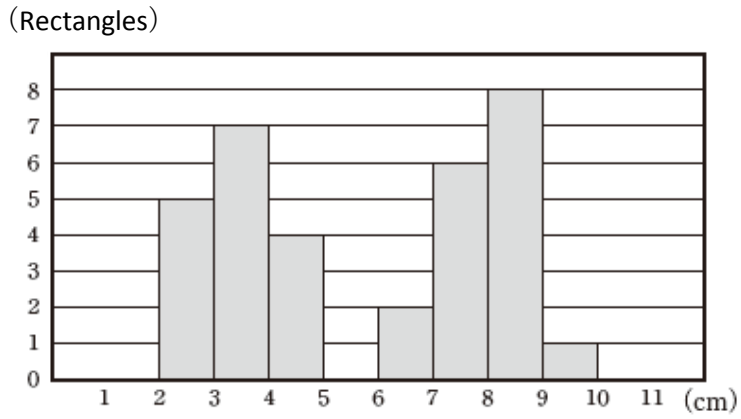
[5] Mai and Koharu decided to investigate which rectangles their classmates consider beautiful. So, they distributed a survey sheet with a 5 cm segment as shown below to their 33 classmates and asked them to draw a rectangle.

Figure 1 summarizes the results of the survey. From this histogram, we can tell, for example, there were 5 rectangles with the horizontal length (width) greater than or equal to 2 cm but less than 3 cm.

Request for Participation

Please draw a rectangle that you would consider beautiful using the given segment below as a side.

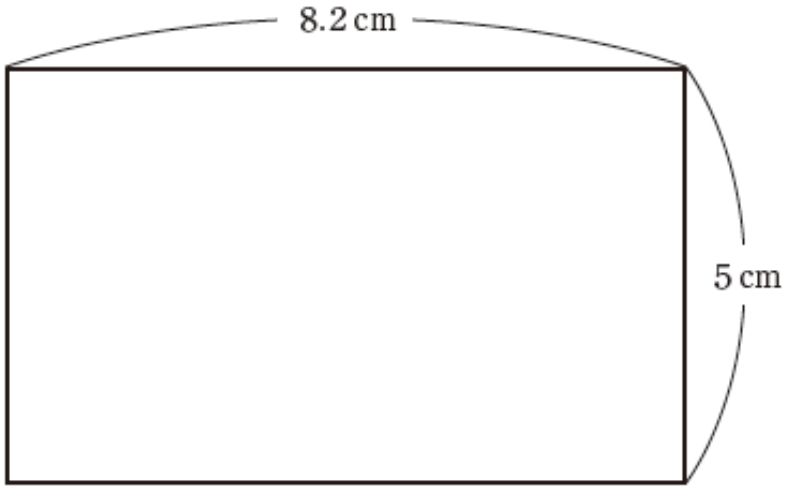
Figure 1: Distribution of Rectangles (Horizontal Length)



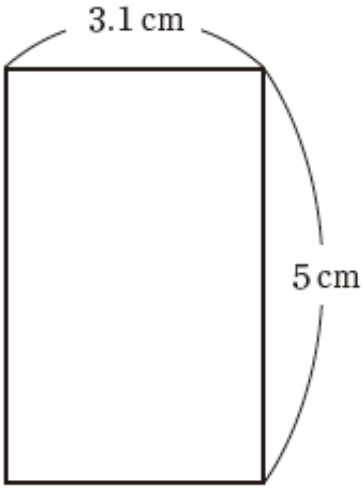
Answer the following questions (1) through (3).

(1) The rectangle Mai drew has the width of 8.2 cm. In Figure 1, it belongs to the category of rectangles greater than or equal to 8 cm but less than 8 cm. The drawing Koharu drew had the width of 3.1cm. To which category in Figure 1 does Koharu's rectangle belong?

Mai's rectangle



Koharu's rectangle



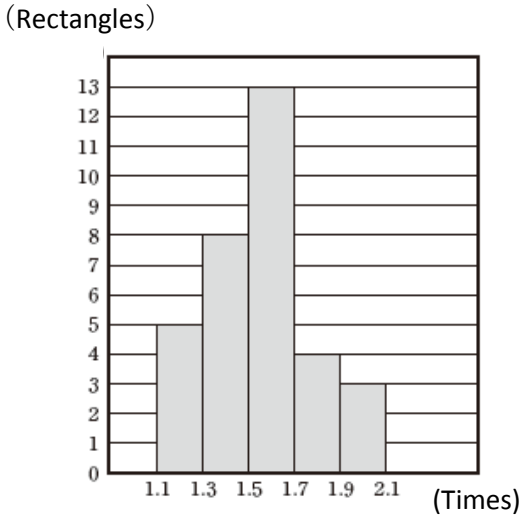
(2) Mai thought if Koharu's rectangle were rotated, it would look similar to the one she drew herself.

Therefore, she determined how many times as long is the longer side of a rectangle as the shorter side on all of the rectangles they collected, and she summarized the results in the histogram shown in Figure 2.

By analyzing the data this way, what new information becomes clearer about which rectangles these students consider beautiful? Explain your answer using the characteristics of the histogram shown in Figure 2.

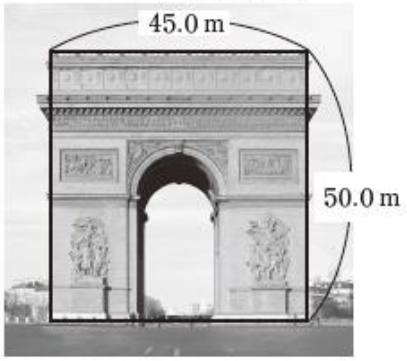
Figure 2

Distribution of Rectangles (Proportion)

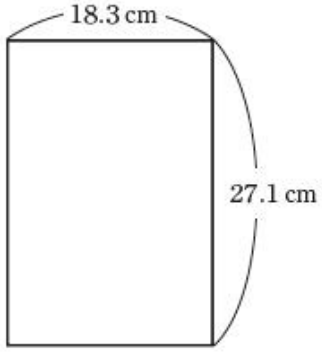


(3) If we consider the shapes of items (a) through (d) as rectangles, there is one that would be included in the category with the highest frequency shown in the histogram in Figure 2. Select the correct one.

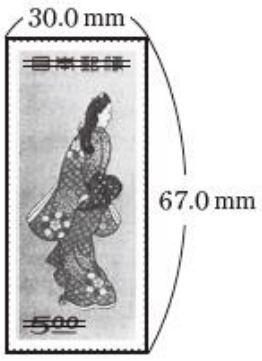
(a) Arc de Triomphe



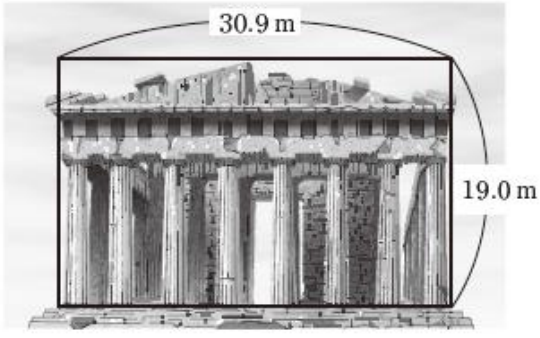
(b) Book of "Taketori Monogatari"



(c) Stamp of "Mikaeri

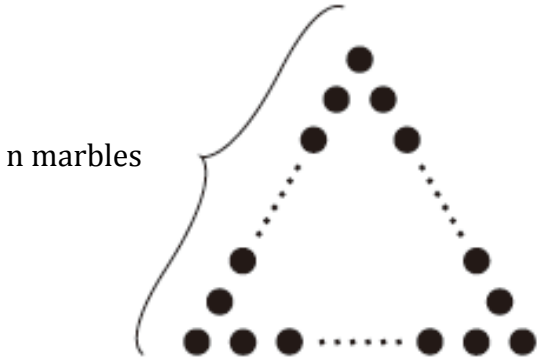


(d) Parthenon Temple



[6] As shown in Figure 1, marbles are arranged in an equilateral triangle formation with n marbles on each side, and the total number of marbles is to be determined.

Figure 1



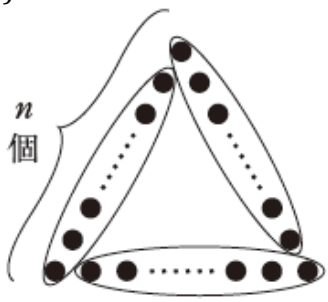
Answer the following questions (1) through (3).

(1) Determine the total number of marbles when marbles are arranged in an equilateral triangle formation with 5 marbles on each side.

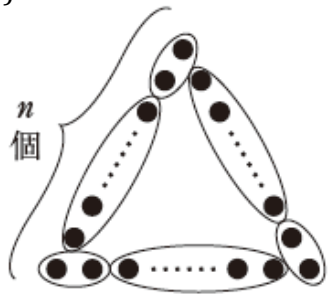
(2) In Figure 1, if we group marbles in a certain way, we can see that the total number of marbles can be expressed as 3(n-1). In (a) through (d) below, such a grouping is shown. Select the correct one.

(a)

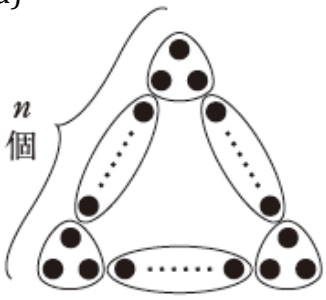
(b)



(c)

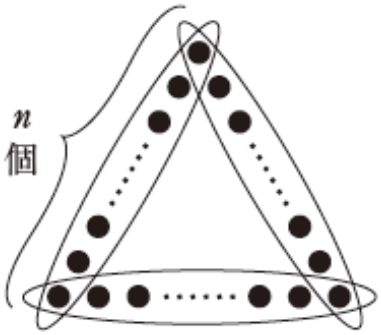


(d)



(3) If we group the marbles as shown in Figure 2, the total number of the marbles can be expressed as 3n – 3. We can explain the reason why the expression for the total number of marbles is 3n - 3 as follows.

Figure 2



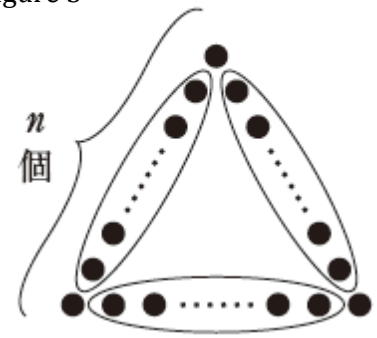
Explanation

Since the marbles on each side of the equilateral triangles are grouped, the number of marbles in each group is n. Since there are 3 equal groups, the total number of marbles in these groups will be 3n. However, since the marble at each vertex is counted twice, the total number of marbles should be 3 less than 3n.

Therefore, the expression for the total number of marbles will be 3n – 3.

If we change the way of grouping these marbles to the one shown in Figure 3, the expression for the total number of marbles will be 3(n - 2) + 3. Complete the explanation below showing the reason why the total number of marbles can be expressed as 3(n - 2) + 3.

Figure 3



Explanation

Therefore, the expression for the total number of marbles will be 3(n – 2) + 3.